Nonlinear Dynamic Response of Unanchored Ground-Based Tanks

Ph.D. Proposal by

Al Zeiny

Committee in charge:
Professor Medhat A. Haroun, Chair
Professor Jann-Nan Yang
Professor Jose A. Pires
Professor Roberto Villaverde
Professor George Lueker

SUMMARY

Under horizontal earthquake excitations, the overturning moment exerted from the hydrodynamic pressures on unanchored, thin-walled liquid storage tanks tends to lift the shell off its foundation, thus developing a nonlinear uplift problem. Additional nonlinearities involved in the analysis of such tanks are due to the large deformation of the base plate, pre and post-buckling behavior of the shell, material yielding, soil plasticity, and large-amplitude free surface sloshing. It is proposed to develop a three-dimensional finite element program on the Convex C240 mini-super computer to obtain the dynamic response of these tanks to a given base excitation taking into consideration the aforementioned nonlinearities.

1 Introduction

Liquid storage tanks are important components of lifeline and industrial facilities. They are critical elements in municipal water supply and fire-fighting systems, and in many industrial facilities for storage of water, oil, chemicals and liquefied natural gas. The damage of large tanks during seismic events has implications far beyond the mere economic value of the tanks and their contents. If, for instance, a water tank collapses, as occurred during the 1933 Long Beach and the 1971 San Fernando earthquakes, loss of public water supply can have serious consequences. Similarly, failure of tanks storing combustible materials, as occurred during the 1964 Niigata, Japan and the 1964 Alaska earthquakes, can lead to extensive uncontrolled fires. Thus, earthquake forces generated by ground motions should be considered in the design of these tanks.

Steel ground-based tanks consist essentially of a steel shell that resists the outward liquid pressure, a thin flat bottom plate that prevents the liquid from leaking out, and a thin roof plate that protects the content from the atmosphere. It is common to classify such tanks in two categories depending on the support condition: anchored and unanchored tanks. Anchored tanks must be connected to large foundations to prevent uplift in the event of earthquake occurrence. It is common, particularly for large size tanks, to support the shell on a ring wall foundation without anchor bolts and to support the bottom plate on a compacted soil though, sometimes, ring walls are omitted. Circular vertical tanks are more numerous than any other type because they are efficient in resisting the liquid hydrostatic pressure by membrane stresses, simple in design, and easy in construction.

Most of the tanks in the field are essentially unanchored. Although several methods and techniques are provided by different standards and codes for mitigating seismic hazards on unanchored liquid storage tanks, the performance of such tanks during past earthquakes revealed a much more complex behavior than is implied by current design procedures and demonstrated the need for reliable analyses to assess their seismic safety. For example, the calculation of the earthquake induced loads is much more complicated due to the fact that the base of these tanks lift off its foundation, if the ground shaking is sufficiently strong. For an anchored tank, it is postulated that the shell of such tanks vibrates axisymmetrically under vertical excitations and asymmetrically under horizontal excitations. The behavior of unanchored tanks is totally different since assumptions of symmetry and asymmetry of the tank shell are no longer valid because of the tank separation from its foundation. Hence, one must investigate a nonlinear liquid-structure-soil interaction problem. For such tanks, overturning moment caused by the hydrodynamic pressure tends to lift shell off its foundation. As the shell displaces upward, it pulls against the tank bottom causing a "crescent-shaped" region of the base plate to lift off soil. The weight of the tank contents resting on the uplifted area of the plate provides resistance to the upward shell movement and prevents further uplift. As a result of tank uplift, the junction between the bottom plate and the shell may experience cracks. On the opposite side, high compressive stresses could develop causing buckling of the shell. The combined actions of liquid sloshing and of accelerating liquid cause the tank to rock, and in time, this rocking motion may spread buckles around the bottom of the tank. In addition, if amplitudes of liquid sloshing motion is greater than the allowed freeboard, serious damage to the roof-shell connection, buckling of the shell at tank top and buckling of the roof plate may occur.

Improved methods of analysis are needed to properly account for the effects of large amplitude base uplifting and of large amplitude liquid sloshing. It is important to reproduce the complex nonlinear response mechanisms revealed in past seismic events with a reasonable degree of accuracy.

2 Justification of the Current Work

The dynamic behavior of liquid storage tanks has been the subject of numerous research during the past twenty years. Theoretical and experimental investigations have been conducted to seek possible improvements in the design of such tanks to resist earthquakes. Initial research on the subject of dynamic response of tanks started in the late 1940s in the field of aerospace technology. The emphasis of those studies was on the influence of the vibrational characteristics of liquid containers on the flight control system of space vehicles (e.g. [1]).

Numerous studies were performed to investigate the seismic behavior of unanchored tanks. Because of the complexity of the problem, most of these studies were experimental in nature ([6], [7], [8], [9], [10], [11], [30], [31], [39], [40], [51], [52]). Some of these tests were performed on full scale tanks such as in [48]. In addition, several simplified theoretical investigations were conducted ([2], [3], [4], [5], [7], [8], [19], [28], [29], [33], [37], [41], [43], [46], [47], [53]). A few of these studies have been used as a basis for current design standards. The large-scale damage to unanchored tanks in recent earthquakes highlighted the need for a careful analysis of such tanks.

The numerical discretization approach, such as the finite element and the finite difference techniques, was employed ([2], [4], [5], [24], [36], [38], [44], [54]) to analyze unanchored tanks. However, approximate assumptions were made to simplify the analysis, such as the substitution of the base plate by "equivalent" springs, the performing of a pseudo-dynamic analysis in lieu of the full dynamic analysis, the linearization of a portion of the problem such as considering the tank wall to be rigid or ignoring large amplitude liquid sloshing, and the use of approximate analytical expressions for the hydrodynamic pressure to eliminate the liquid degrees of freedom.

The next generation of analysis to obtain the most accurate results must consider a nonlinear dynamic response to account for material, liquid, tensionless foundation, slippage and buckling nonlinearities.

3 Research Outlines

3.1 The Objectives

The following responses to a given deterministic base excitation shall be obtained from the analysis:

1. The hydrodynamic pressure distribution on the tank shell.
2. The deformation of the tank shell.
3. The overturning moment and the base shear exerted on the tank.
4. The stresses in both the tank shell and the base plate.
5. The uplift displacement and the boundary of the contact area.
6. The liquid sloshing amplitude.
7. The stress distribution in the underlying soil.

3.2 Modeling of the problem

Earthquakes are caused by a sudden energy release in a volume of rock lying on a fault. This source is normally located a large distance away and at a significant depth from the site. Even if all details of how the source mechanism works and the data on the travel path of the seismic waves to the site were available (which is, of course, not the case) it would still be impossible to model all aspects of the problem because of the size of the dimensions compared to those of the tank. In any event, the many uncertainties involved make it meaningless to analyze the complete earthquake-excitation problem. In this research, the prescribed ground motion is considered as being derived from an observed record at this very site, or at least a similar site, and the corresponding tank-soil-liquid system response is found.

The entire problem is then modeled by disceretizing the liquid, tank and soil domains by three-dimensional finite elements. The coordinate of a generic point in the space is defined by a cylindrical coordinate system.

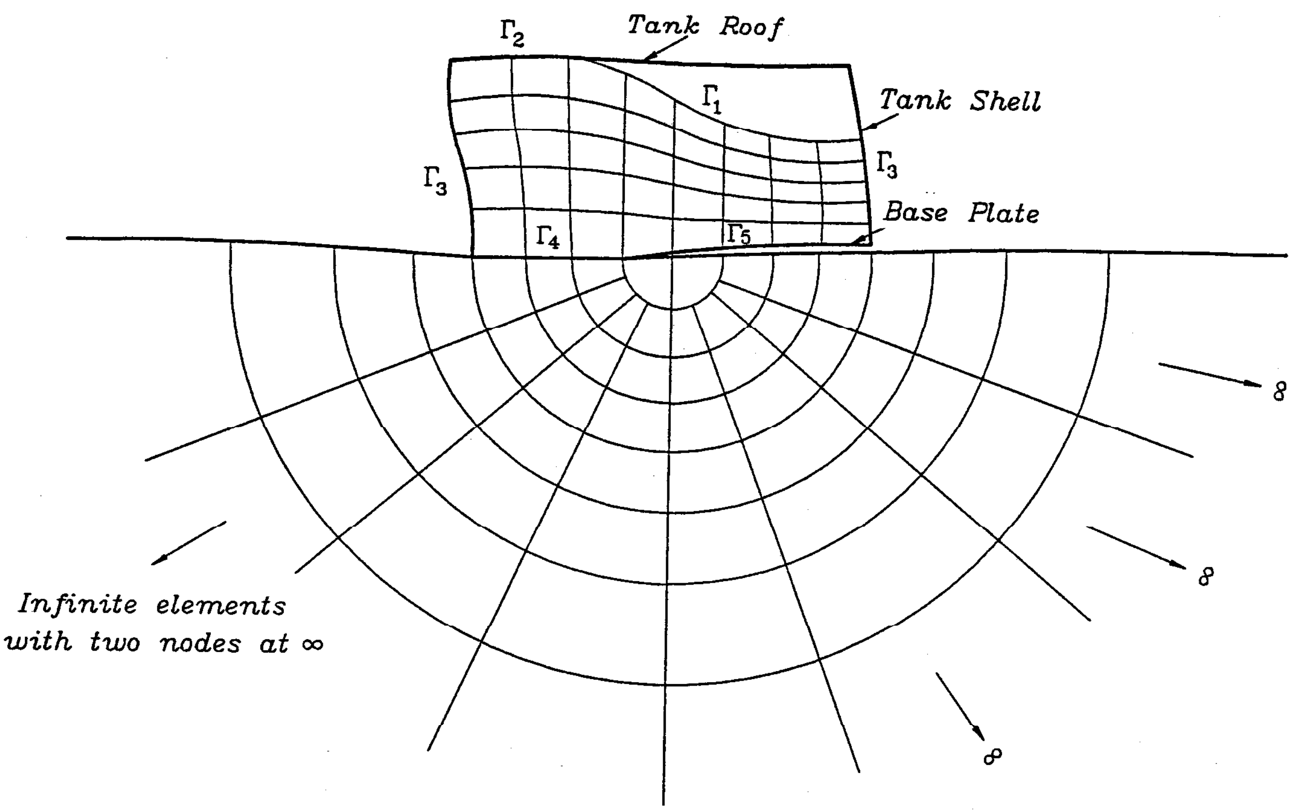

Figure 1: Outline of the mesh and the boundaries.

In order to reduce the finite element modeling error, a nonlinear three dimensional eight-noded liquid element with two circular faces will be developed to model the liquid in circular tanks. In addition, a nonlinear three dimensional four-noded circular shell element will also be developed to model the circular shell. Using these elements, the number of elements required to fit the circle will be substantially reduced from that using isoparametric elements. Consequently, this would allow more efficient use of the computer memory and reduce the program execution time.

Alternatively, the isoparametric nonlinear liquid brick element along with the nonlinear isoparametric curved shell elements may be used. In any of these cases an isoparametric nonlinear soil element should be used to model the soil. The soil is semi-infinite medium, an unbounded domain. A fictitious boundary at a sufficient distance from the structure, where the response is expected to have died out from a practical point of view, would reflect waves originating from the vibrating structure back into the discretized soil region instead of letting them pass through and propagates toward infinity. In order to avoid this, a finite element with two nodes at infinity should be used. The prescribed base excitation is then applied at the soil surface.

3.3 Nonlinearities Arising from Unknown Boundaries

To a large extent, the complexity of the problem arises mostly form the unknown boundaries of the liquid. As shown in Fig. (1), the free surface Γ_1 varies at all times and may be ultimately bound by the roof surface Γ_2. The liquid inside the shell is bounded by the vibrating shell surface Γ_3 and the base plate which by itself is in a successive contact with and separation from the soil at Γ_4 and Γ_5, respectively. This successive contact and separation of the base plate will induce friction and slippage forces that must be considered in addition to the large deformation for the base plate. None of the boundaries will keep its shape as time passes on.

3.3.1 The Unknown Free Surface Elevation

The elevation of the free surface is basically unknown and varies with time. This would require an updating of the mesh after each time step. According to the following equation, the free surface elevation at specific node at any time step may be extrapolated from the previous time steps

$$\eta(t+\Delta t) = \eta(t) + \Delta t \frac{d\eta}{dt} + \frac{\Delta t^2}{2}\frac{d^2\eta}{dt^2} + \cdots \tag{1}$$

where η is the free surface elevation, t is the time and Δt is the time step.

Updating of the mesh will be performed in such a way that the shape of the horizontal mesh lines gradually changes from the shape of the free surface to the shape of the tank base plate (Fig. 1). In the case of insufficient freeboard, the free surface may pound the roof. In such a case, Γ_2 will be considered an extension of Γ_3 except for the impact of the liquid.

3.3.2 The Contact Area Nonlinearity

Since the tank is unanchored and the foundation is tensionless, successive contact and separation between the base plate and its foundation, and slippage may take place during strong ground motion.

There is a pair of nodes on and under the base plate. One node belongs to the soil while the other belongs to the base plate. The two nodes are connected by a gap element Iterative solution will be applied based on opening the gap element that have tension force F_v and allowing a relative slippage according to the ratio of the horizontal reaction at the base plate node to the vertical one (F_h/F_v) when F_v is compression until the equilibrium condition is satisfied. The flow chart shown in Fig. (2) outlines the mechanism of the gap element.

The impact between the soil and the tank is considered by applying abrupt coupling between the base plate node and the soil node when they get in contact.

3.4 Nonlinearities Arising From the Governing Differential Equations

For a general boundary for which the kinematic equation is $F(r, \theta, z, t)$ = const, the general liquid normal velocity condition at this boundary is

$$\begin{aligned} \frac{dF}{dt} &= \frac{\partial F}{\partial t} + v_r \frac{\partial F}{\partial r} + \frac{v_\theta}{r}\frac{\partial F}{\partial \theta} + v_z \frac{\partial F}{\partial z} \\ &= \frac{\partial F}{\partial t} + \vec{V}.\vec{\nabla} F \qquad (2) \\ &= \frac{\partial F}{\partial t} + \vec{\nabla}\Phi.\vec{\nabla} F = 0 \end{aligned}$$

where v_r, v_θ and v_z are the boundary velocity components in the cylindrical coordinate system, ∇F is the boundary gradients, $\vec{V}$ is the velocity vector and $\vec{\nabla}$ is the gradient vector in the cylindrical coordinate system. The variation of the boundary gradients for the shell and base plate with time is negligible and the normal to the boundaries at any time may be considered to have the same direction.

The strong form of the governing equations for unsteady incompressible liquid under the potential flow conditions may be stated as follows:

Given the initial conditions of the potential function $\Phi_o(r, \theta, z)$ and the initial boundaries Γ_o, find $\Phi(r, \theta, z, t)$: $\overline{\Omega} \times [0, T] \rightarrow \mathbf{R}$ and $\Gamma(t)$ such that

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \Phi}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \qquad (3)$$

$$\Gamma(0) = \Gamma_o$$

$$\Phi(r, \theta, z, 0) = \Phi_o(r, \theta, z) \ on \ \Omega$$

$$\frac{\partial \Phi}{\partial z} = \frac{d\eta}{dt} \ on \ \Gamma_1 \qquad (4)$$

$$\frac{\partial \Phi}{\partial t} = -\frac{\nabla\Phi \nabla \Phi}{2} - g\eta \ on \ \Gamma_1 \qquad (5)$$

$$\frac{\partial \Phi}{\partial r} = \frac{\partial w_r}{\partial t} \ on \ \Gamma_3$$

$$\frac{\partial \Phi}{\partial z} = \frac{\partial w_z}{\partial t} \ on \ \Gamma_2, \ \Gamma_4 \ and \ \Gamma_5$$

where w_r is the tank shell deflection in the r direction, w_z is the tank base plate and roof deflection in the z direction, $\Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup \Gamma_4 \cup \Gamma_5$ and r, θ and z are the cylindrical coordinate of a generic point.

The weak (variational) form of the liquid problem may be stated as follows: Given $\Phi_o(r, \theta, z)$ and Γ_o, find $\Phi(r, \theta, z, t) \in \delta_t$, t $\in$ [0,T] such that $\forall\, \overline{w} \in \mho$

$$\int_\Omega \nabla \overline{w} \nabla \Phi d\Omega - \int_{\Gamma_1} w_1 \left[\frac{\partial \Phi}{\partial t} + \frac{\nabla\Phi \nabla \Phi}{2} + \alpha(\eta) + \alpha_l(\eta)\frac{\partial \Phi}{\partial z} + \alpha_m(\eta)\frac{\partial \Phi}{\partial \theta} + \alpha_n(\eta)\frac{\partial \Phi}{\partial r}\right] d\Gamma$$

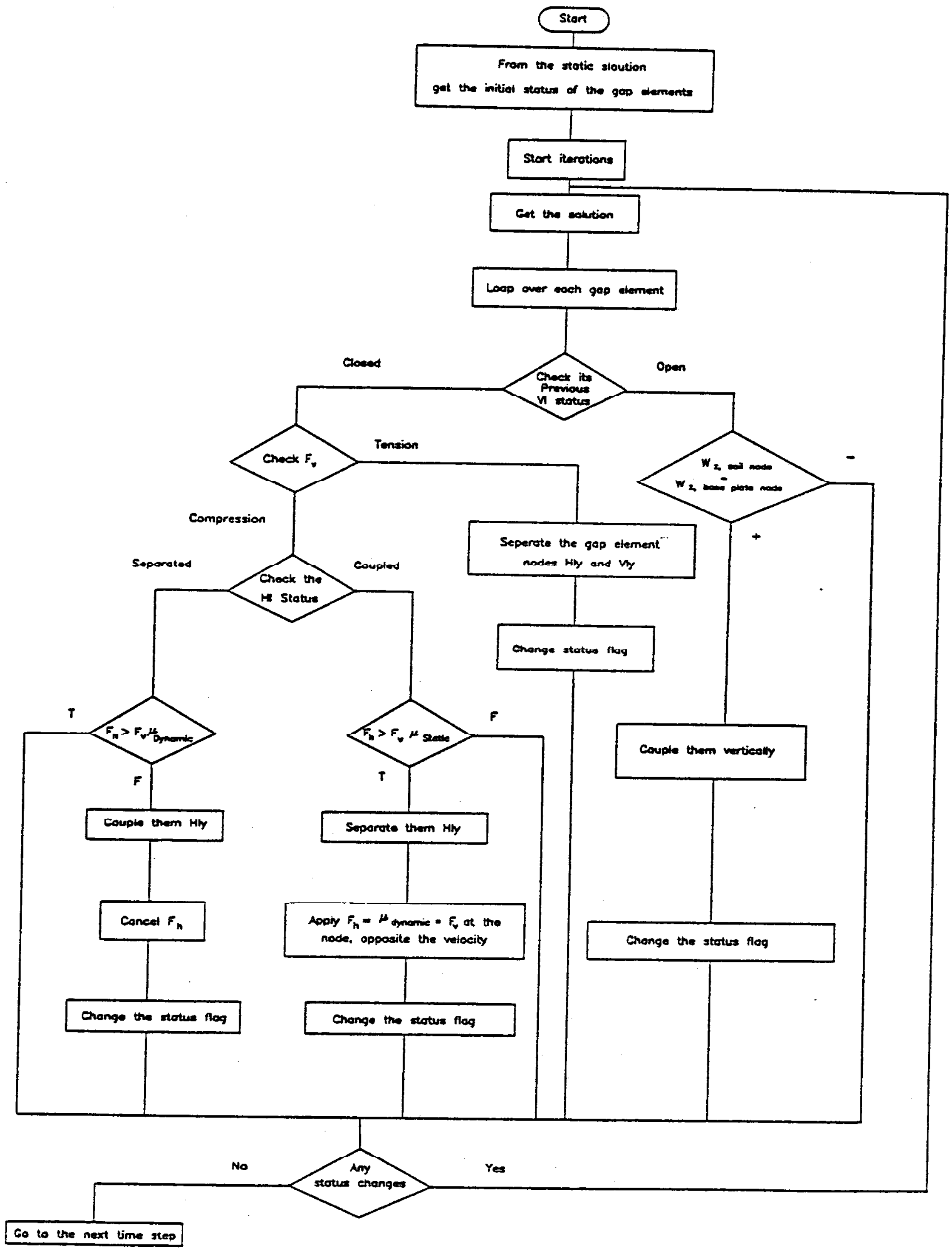

Figure 2: Flow chart to outline modeling of the contact area by a gap element.

$$= \int_{\Gamma_2,\Gamma_4,\Gamma_5} \overline{w_1}\frac{\partial w_z}{\partial t} d\Gamma + \int_{\Gamma_3} \overline{w_1}\frac{\partial w_r}{\partial r} d\Gamma \tag{6}$$

$$\alpha(\eta) = g\eta - \frac{\partial \eta}{\partial t} \tag{7}$$

$$\alpha_l(\eta) = 1 + \cos\alpha\cos\beta \tag{8}$$

$$\alpha_m(\eta) = \frac{\cos\alpha\sin\beta}{r} - \frac{1}{r^2}\frac{\partial\eta}{\partial\theta} \tag{9}$$

$$\alpha_n(\eta) = -\sin\alpha\cos\beta - \frac{\partial\eta}{\partial r} \tag{10}$$

$$\alpha = \tan^{-1}(\frac{\partial\eta}{\partial r}) \tag{11}$$

$$\beta = \tan^{-1}(\frac{1}{r}\frac{\partial\eta}{\partial\theta}) \tag{12}$$

where δ_t is the space of trial solutions, $\mho$ is the space of the weighting functions which satisfy the essential boundary conditions, w_1 and $\overline{w_1}$ are the boundaries of $\overline{w}$ on Γ_1 and $\overline{\Gamma_1}$, respectively, and $\overline{\Gamma_1} = \Gamma - \Gamma_2 - \Gamma_3 - \Gamma_4 - \Gamma_5$.

3.4.1 The Free Surface Boundary Conditions

- **The Kinematic Condition**

A liquid particle on the free surface should have the same velocity as the free surface. Using Eq. (4), this condition may be rewritten as follows

$$\frac{\partial\Phi}{\partial z} = \frac{\partial\eta}{\partial t} + \frac{\partial\eta}{\partial r}\frac{\partial\Phi}{\partial r} + \frac{1}{r^2}\frac{\partial\eta}{\partial\theta}\frac{\partial\Phi}{\partial\theta} \tag{13}$$

This relation includes the free surface gradients. Those are unknowns and need to be extrapolated from the previous time steps and corrected after each iteration until the equilibrium condition is satisfied.

- **The Dynamic Condition**

Equation (5) represents the dynamic condition written with the aid of Bernulli's equation. The value of $\partial\Phi/\partial t$ at the free surface is nonlinear function of Φ and should be extrapolated from the previous time steps and corrected after each iteration until the equilibrium condition is satisfied.

3.4.2 Large Deformation of Base Plate and Shell Buckling

The uplift of the base plate could be of the order 2–14 inches. Since this value is greater than the base plate thickness, the small deformation theory is no longer valid. The bending deformations stretch the base plate and engage the membrane stiffness. A nonlinear shell element is used to model this phenomenon. The relations between strains and deformations in a flat base plate and a flat roof taken into consideration the base plate large deformations and roof buckling is given in terms of the normal and shear strains in the middle surface ϵ_r, ϵ_θ and $\epsilon_{r\theta}$, in terms of mid surface change of curvature K_r and K_θ, and in terms of the midsurface twist $K_{r\theta}$ as follows

$$
\begin{aligned}
e_r &= \epsilon_r + x_t K_r && (14)\\
e_\theta &= \epsilon_\theta + x_t K_\theta && (15)\\
\gamma_{r\theta} &= \epsilon_{r\theta} + x_t K_{r\theta} && (16)\\
\epsilon_r &= \frac{\partial w_r}{\partial r} \pm \frac{1}{2}(\frac{\partial w_z}{\partial r})^2 && (17)\\
\epsilon_\theta &= \frac{w_r}{r} + \frac{1}{r}\frac{\partial w_\theta}{\partial \theta} \pm \frac{1}{2}(\frac{1}{r}\frac{\partial w_z}{\partial \theta})^2 && (18)\\
\epsilon_{r\theta} &= \frac{1}{r}\frac{\partial w_r}{\partial \theta} + \frac{\partial w_\theta}{\partial r} - \frac{w_\theta}{r} \pm \frac{1}{r}\frac{\partial w_z}{\partial r}\frac{\partial w_z}{\partial \theta} && (19)\\
K_r &= -\frac{\partial^2 w_z}{\partial r^2} && (20)\\
K_\theta &= -\frac{1}{r}(\frac{\partial w_z}{\partial r} + \frac{1}{r}\frac{\partial^2 w_z}{\partial \theta^2}) && (21)\\
K_{r\theta} &= \frac{2}{r}(\frac{1}{r}\frac{\partial w_z}{\partial \theta} - \frac{\partial^2 w_z}{\partial \theta \partial r}) && (22)
\end{aligned}
$$

where x_t is the distance of a generic point measured from the midsurface in a direction normal to it and w_z, w_θ and w_r are tank base plate deflections in the cylindrical coordinate system. The positive sign is used to include the roof buckling while the negative one is used to include the base plate large deformation.

Since the tank shell may buckle, the shell element should also account for buckling as well as large deformation. The relations between strains and deformations in the tank shell taken into consideration the shell buckling is given in terms of the normal and shear in the middle surface ϵ_z, ϵ_θ and $\epsilon_{z\theta}$, in terms of mid surface change of curvature K_z and K_θ, and in terms of the midsurface twist $K_{z\theta}$ as follows

$$
\begin{aligned}
e_z &= \epsilon_z + x_t K_z && (23)\\
e_\theta &= \frac{1}{(1+\frac{x_t}{R})}(\epsilon_\theta + x_t K_\theta) && (24)\\
\gamma_{z\theta} &= \frac{1}{(1+\frac{x_t}{R})}\left(\epsilon_{z\theta} + x_t(1+\frac{x_t}{2R})K_{z\theta}\right) && (25)\\
\epsilon_z &= \frac{\partial w_z}{\partial z} + \frac{1}{2}(\frac{\partial w_r}{\partial z})^2 && (26)\\
\epsilon_\theta &= \frac{1}{R}(\frac{\partial w_\theta}{\partial \theta} + w_r) + \frac{1}{2R}(\frac{\partial w_r}{\partial \theta} - w_\theta)^2 && (27)
\end{aligned}
$$

$$\epsilon_{z\theta} = \frac{1}{R}\frac{\partial w_z}{\partial \theta} + \frac{\partial w_\theta}{\partial z} + \frac{1}{R}\frac{\partial w_r}{\partial z}(\frac{\partial w_r}{\partial \theta} - \frac{w_\theta}{R}) \quad (28)$$

$$K_z = -\frac{\partial^2 w_r}{\partial z^2} \quad (29)$$

$$K_\theta = -\frac{1}{R^2}(\frac{\partial^2 w_r}{\partial \theta^2} - \frac{\partial w_\theta}{\partial \theta}) \quad (30)$$

$$K_{z\theta} = -\frac{2}{R}(\frac{\partial^2 w_r}{\partial \theta \partial z} - \frac{\partial w_\theta}{\partial z}) \quad (31)$$

where R is the tank radius, and w_z, w_θ and w_r are tank sell deflections in the cylindrical coordinate system.

3.4.3 Material Nonlinearity

Previous experience of tank behavior under strong ground motion showed that the tank may undergo plastic deformations. Also, under the seismic loads, hysteretic loops may be developed. A nonlinear material model will be used to include this effect.

It should be noted that the value of the shell membrane stresses σ_r and σ_θ are different. This will yield unequal E_r and E_θ and the material nonlinearity will develop anisotropy in the tank shell which should be accounted for in the development of the shell element.

3.4.4 Coupling of the Nonlinear Shell, Tank and Soil Matrix Equations

The nonlinear matrix equation for the tank in the time domain is

$$[M]\{\ddot{d}\} + [C]\{\dot{d}\} + ([K] \pm [K_G])\{d\} = \{F_e\} + \{P_s\} + \{P_d\} \quad (32)$$

where $[K_G]$ is the nonlinear geometric matrix which takes the positive sign in the case of large base plate deformation and the negative one in the case of shell buckling, $\{P_s\}$ and $\{P_d\}$ are the nodal force vector on the tank due to the liquid static and dynamic pressures, respectively, and $\{F_e\}$ is the equivalent nodal force on the tank due to base excitation. The liquid matrix equation in the time domain is

$$[M_f]\{\dot{\Phi}\} + [K_f]\{\Phi\} = \{V_t\} + \{G\} \quad (33)$$

where $[M_f]$ is the liquid mass matrix at the free surface nodes, $[K_f]$ is the liquid Stiffness matrix, $\{V_t\}$ = liquid force vector due to shell, roof and base plate normal velocities, $\{G\}$ is the nonlinear part of the force vector arising from the free surface large oscillations. The soil matrix equation in the time domain is

$$[M_s]\{\ddot{d}\} + [C_s]\{\dot{d}\} + [K_s]\{d\} = \{F_{es}\} + \{R_t\} \quad (34)$$

where $[M_s]$ is the soil mass matrix, $[C_s]$ is the soil viscous damping matrix, $[K_s]$ is the soil static stiffness matrix, $\{F_{es}\}$ is the equivalent nodal forces on the soil due to the ground motion and $\{R_t\}$ is the tank reaction. The vectors $\{F_{es}\}$ and $\{R_t\}$ represent inertial soil-structure interaction part while $\{F_e\}$ represent the kinematic soil tank interaction.

The three matrix equations should be solved simultaneously. The vector $\{P_d\}$ should be found by applying Bernulli's equation at the boundary nodes between the liquid and the tank and $\{V_t\}$ should be extrapolated from the previous time steps and corrected after each iteration until the equilibrium condition is satisfied.

4 Outcome

This work will improve the state of knowledge of the behavior of unanchored ground-based liquid storage tanks during ground motion and the nature of large-amplitude shell uplift and liquid sloshing. It will also, add two new elements to the finite element library as well as emphasize the usefulness of combining the soil, shell and liquid elements together, and introduce the use of infinite soil element to avoid a reflecting fictitious soil boundary. In conclusion, the outlined approach will provide a better prediction of the response of the tank system to a given base excitation which may then be used to improve the current design practice and mitigate seismic hazards to these important structures.

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